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Review text:

Hestenes' explanation of a new gauge theory of gravity, due to the Cambr. Univ. Geom. Alg. Res. Group [Doran, C., Lasenby, A. (2003), Geom. Alg. f. Phys., CUP] suits graduate students and researchers in theor. and math. physics, with interests in differential geometry, (gauge) theories of gravity, and unification with electrodynamics and quantum theory. It can be read on its own, but it is recommended to also study [Hestenes, D. (2003), Oersted Medal Lect. 2002: Reforming the math. lang. of phys., Am. J. Phys. 71: 104-121] and to know basic general relativity.

The introduction gives an overview of concepts involved and reasons for the use of geometric calculus based on (Clifford) geometric algebra of space-time (STA, isomorphic to $Cl_{1,3}$), with invariant coordinate free multivector calculations.

Section two introduces STA, which reinterprets Dirac matrices as an orthonormal vector basis of the real 4D Minkowski vector space. Lorentz rotations are implemented by the rotor (spin) group SU(2). (Proper time) parametrized rotors express particle dynamics by a generalized rotational bivector velocity and define comoving frames. The geometric calculus vector derivative allows to give real multivector forms to Maxwell's equations (in *one* equation) and the Dirac equation. Section three applies geometric calculus to reshape linear algebra and differential transformations free of matrices and coordinates.

The central section four employs a flat space model, which identifies the spacetime manifold with the Minkowski vector space. A spacetime map describes the spaciotemporal partial ordering of physical events as points in the spacetime manifold. Fields on spacetime take values in the flat tangent space algebra. The displacement gauge principle (DGP) that *equations of physics must be invariant under arbitrary smooth remappings of events on spacetime* and the rotation gauge principle (RGP) of *covariance under (active) local Lorentz rotations* are introduced. They correspond to globally homogeneous (DGP) and locally isotropic (RGP) spacetime.

The DGP is realized with a new physical (gravitational) field, an invertible gauge field (or tensor), equivalent to a *tetrad field*. This leads to position gauge *invariant* forms of (vector) derivative, velocity, and the line element with symmetric metric tensor; and completely decouples the remapping of events in spacetime from coordinate changes. The DGP displacement *symmetry* group leaves the flat spacetime background invariant.

The RGP leads to a gauge covariant derivative (vector coderivative) including a bivector valued connexion tensor. Torsion free Riemannian geometry is assumed. The commutator of the coderivatives defines the curvature, which can be expressed as a covariant bivector-valued function of a bivector variable. Derivations in completely coordinate free form are given for curvature, curvature contractions, coderivative identities and Bianchi identities.

Section five successively formulates Einstein's equation, electrodynamics with gravity, the derivation of equations of motion from Einstein's equation, particle motion, parallel transfer, gravitational precession and the real Dirac equation with gravitational interaction. Einstein's tensor is given a new unitary form. In section six very compact multivector solutions for the curvature tensors of static and rotating (Kerr) black holes in various gauges are given and applied to elementary black hole physics and (gravitational) orbital precession.

In section seven a position gauge invariant *canonical energy-momentum split* of Einstein's equations allows to identify *total* and *gravitational field* energy-momentum tensor densities. The total density is the divergence of a bivector valued *superpotential* (function of the vector connexion). We therefore have a coordinate independent energy-momentum tensor. There is a rotation gauge dependence, but superpotential and energy-momentum tensor preserve local gauge equivalence.

Section eight proposes six general laws of spacetime structure and measurement including gauge equivalence and energy-momentum conservation, modeled on Newtonian mechanics. Relativistic quantum mechanics is included (e.g. via a Lagrangian approach). Einstein's weak and strong principles of equivalence are related to the DGP and RGP. Newton's first law too is identified as a principle of gauge equivalence. Weak and strong interactions are not treated, but it is wellknown that Clifford algebra is essential in relativistic quantum field theory.

Comments to the MR Editors: Sorry for the late submission.