This is a review text file submitted electronically to MR.

Reviewer: Hitzer, Eckhard M. S.

Reviewer number: 036883

Address:

Department of Physical Engineering, University of Fukui, 3-9-1 Bunkyo, Fukui, Fukui 910-8507, Japan
hitzer@mech.fukui-u.ac.jp

Author: Reifler, Frank; Morris, Randall

Short title: Hestenes' tetrad and spin connections.

MR Number: 2199619

Primary classification: 81R25

Secondary classification(s): 81T13, 34L40, 15A66

Review text:

This paper suits graduate students and researchers in gauge field theory, quantum theory, general relativity and its generalizations. The derivations are step by step in a good educational style.

Hestenes [*Real Spinor Fields*, J. of Math. Phys., 8, pp. 559–649, 1967] showed using Clifford geometric algebra that a bispinor field on a Minkowski space-time is equivalent to an orthonormal tetrad of vector fields and a complex scalar field. Hestenes' tetrad allows to show that the Dirac bispinor Lagrangian depends only on these fields. In all of space-time the gravitational field (metric) and the (energy bounded) bispinor field are represented accurately by Hestenes' tetrad and complex scalar fields. This leads to simplifications and new solutions.

The paper endeavors to give a straightforward derivation of the *covariant* tensor form of the Dirac Lagrangian using Hestenes' tetrad as reference for the spin connection in Riemannian space-time. The tetrad allows to define the spin connection acting on a bispinor field. With the spin connection Dirac's bispinor Lagrangian in a Riemannian space-time can be given with fermion mass m_0 coupling to a complex scalar field. The $SL(2,R) \times U(1)$ gauge invariance leads to four Noether currents, including electromagnetic and chiral currents. These currents and the complex scalar field satisfy a Fierz identity. Hestenes' orthonormal reference tetrad (local rest frame for the bispinor field) is defined in terms of these currents and the complex scalar. Then the bispinor field locally only depends on the complex scalar.

Reifler and Morris [*Measuring a Kaluza-Klein radius smaller than the Planck length*, Phys. Rev. D 67, 064006, 2003] showed earlier, that in a Kaluza-Klein model based on a constrained Yang-Mills form of Dirac thoery Hestenes' tetrad and complex scalar can be mapped bijectively (via the four Noether currents) to a set of $SL(2,R) \times U(1)$ gauge potentials and another complex scalar field. The Yang-Mills Lagrangian for these gauge potentials and the complex scalar field equals the Dirac bispinor Lagrangian in the limit of an infinitely large coupling constant. The scalar field can be used as a Higgs field for generating the fermion mass. The authors argue for the experimental irrelevance of possible singularities in the tensor field in complements of open dense space-time subsets.

The authors further derive (non-symmetric) energy-momentum and spin polarization tensors in Minkowski and Riemannian space-times in terms of bispinors and gauge tensors.

The last section generalizes the Dirac Lagrangian to arbitrary linear connections. Transition from tensors to bispinors using orthonormal tetrads of vector fields leads to general spin connections acting on bispinors. First torsion-free connections are considered. This leads to a form of the Dirac Lagrangian including the totally antisymmetric spin polarization tensor. The authors use this antisymmetry to derive a Dirac Lagrangian with form invariant Dirac equation, only involving the spin connection.

Next totally antisymmetric torsion is included. The arbitrary linear connection and its related spin connection lead to equal (bispinor and tensor) Lagrangians. The Dirac operators are unique for the above definition of spin connection. For Riemannian connections classical spin connection definitions also lead to form invariant Dirac equations, but not in the case of general linear connections (e.g. when adding a Lortentz four-vector field). Finally the difference (torsion!) to teleparallel spin connections is explained.

Comments to the MR Editors: I am sorry to be so late with the submission.