This is a review text file submitted electronically to MR.

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Short title: N-dimensional geometries generated by hypercomplex numbers.

MR Number: 2236622

Primary classification: 53A35

Secondary classification(s): 30G35, 32B05, 53B21, 11E88

Review text:

This paper is suitable for graduate students and researchers in *hypercomplex* (hc) algebra, Clifford algebra, hc function theory and non-Euclidean geometry, and non-Euclidean differential geometry, including physical applications to field theory.

Section 2 defines hc number algebras (Clifford algebras or subalgebras of Clifford algebras) via their structure constants. The authors emphasize commutative hc numbers, because (p.5) *For distributive systems with the unity the differential and integral calculus does exist only if the systems are commutative* [M. Scheffers, C. R. Acad. Sc. 116, p. 1114 (1893)]. Hc numbers are represented by characteristic (char.) matrices leading to invariant char. determinants (det.) and isomorphic matrix algebras. Associativity is expressed by a structure constant equation. Non-zero hc numbers can still yield divisors of zero (zero char. det.). Like for complex numbers, a char. equation yields (principal) conjugations and a modulus definition. Then the authors illustrate their formalism for 2D non-Euclidean systems (hyperbolic, parabolic and elliptic). They discuss decomposable systems, divisors of zero as zeros of component systems, and analytic functions of decomposable systems.

In section 3 addition and multiplication of hc numbers leads to translations of points associated with components and to linear maps via char. matrices. The linear maps combine to multiplicative groups. Unimodular groups (maps with char. det. one) are compared with (Euclidean) orthogonal groups. New geometries arise in dimension ≥ 3, with the invariants given by algebraic forms (char. det.) of degree N, which can be used to fix a metric. The example of 2D hyperbolic numbers is worked out in detail.

In section 4 generalized Cauchy-Riemann conditions are introduced for generalized holomorphic functions from hc domains (points associated with components) into hc image spaces. All partial derivatives (PDs) can then be expressed as functions of the PDs of just one component. Or equivalently: The Jacobian matrix (transpose) of a hc function has the form of a char. matrix. Again the example of 2D hc numbers is treated in detail.

Section 5 introduces the multiplication tables for commutative hc systems with three and four units. [In tables (38) and (51) top row (and left column) zeros should be corrected to e_2 and e_2, e_2, respectively.] Some discussion of equivalent systems with unit versors, char. det., divisors of zero, exponential and logarithmic functions, and Cauchy-Riemann conditions follows. The case of four units introduces the relevant literature and proposes a generalization of commutative Segre quaternions.

Finally the authors suggest the progression: Solutions of alg. equ. of degree N $\rightarrow$ (commutative) hc numbers $\rightarrow$ multiplicative groups $\rightarrow$ geometries $\rightarrow$ differential geometries.

The appendix shows that N-dim. Riemann-flat spaces with const. connections correspond to commutative hc systems with N units.

Comments to the MR Editors: The English of the article is not perfect, but good enough to be well-understood.