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Short title: Commutative hypercomplex numbers and functions of hypercomplex variable: a matrix study.

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Review text:

This paper is of interest to graduate students and researchers in hypercomplex algebras (including Clifford geometric algebras) and hypercomplex function theory (including Clifford analysis). Application is possible to problems in potential theory and special relativity. Special attention is given to *commutative hypercomplex algebras* (CHA) with unity represented by column vectors and matrices. By adopting a suitable eigenvector representation real power series functions obtain an analytic CHA continuation.

The paper begins by defining the N-dimensional (ND) CHA column vector representation. The structure constant matrix of the CHA product enter the definition of hypercomplex numbers by their *associated matrix* M (transpose of the *characteristic matrix* used in [F. Catoni et al, Adv. Appl. Clifford Algebr. 15 (2005), no. 1]). Elementary CHA operations including associativity conditions are explained in detail. Special attention is paid to divisors of zero, non-zero CHA numbers which result in zero CHA products.

The example of 2D (elliptic, parabolic and hyperbolic) CHA is worked out in detail. Complex numbers turn out to be the canonical elliptic 2D CHA.

Algebraic and spectral properties of the associated matrix representation are studied, giving rise to a *natural* orthonormal eigenvector *representation* (provided all eigenvectors are distinct and non-zero). The modulus of a CHA number is defined as the Nth root of the determinant $|M|$ of M. The natural representation allows to define N-1 *conjugations*, whose products yield again $|M|$.

CHA (matrix) functions are introduced as analytic continuations of real power series, best expressed in the natural CHA representation. The 2D example is worked out in detail (including complex numbers). Total and partial CHA function derivatives, derivative operators, partial derivatives, and derivatives with respect to (w.r.t.) conjugated variables are studied. The latter allows to express the generalization of Cauchy-Riemann conditions for ND CHA functions, leading to the notion of *holomorphic* (holomorphic analytic) CHA functions, examples of which are all analytic CHA continuations of real power series.

Derivatives w.r.t. conjugated variables of holomorphic CHA functions are zero. This leads finally to the real *characteristic equation* for holomorphic CHA functions in the natural representation. In the case of 2D CHA the characteristic equation corresponds to the Laplace equation (canonical elliptic = complex numbers) and the wave equation (canonical hyperpolic numbers).

Comments to the MR Editors: Sorry to be so late again. Merry Christmas!