This is a review text file submitted electronically to MR.

Reviewer: Hitzer, Eckhard M. S.

Reviewer number: 036883

Address:
Department of Physical Engineering
University of Fukui
3-9-1 Bunkyo
Fukui 910-8507
JAPAN
hitzer@mech.fukui-u.ac.jp

Author: de Sabbata, Venzo; Datta, Bidyut Kumar

Short title: Geometric algebra and applications to physics.

MR Number: 2283595

Primary classification: 15A66

Secondary classification(s): 81R25, 58A15, 70S15, 83C45

Review text:

This textbook addresses graduate students and researchers in theoretical physics. Part I (chapters 1 to 6) gives an introduction to *Clifford geometric algebra*, closely modelled on the works of *D. Hestenes* [Hestenes, D. (1966), Space-Time Algebra, Gordon and Breach, New York; Hestenes, D., Sobczyk (1984), Clifford Algebra to Geometric Calculus, Reidel, Boston; Hestenes, D. (1986), New Foundations for Classical Mechanics, Reidel, Boston]. Part I is quite accessible even without prior knowledge of geometric algebra. The emphasize is not on mathematical rigor, but on motivating and preparing the tools for the applications of part II.

Of historical interest is chapter 1, which reviews the concept of *(directed) numbers* beginning in the fourth century B.C. The factorization of bivectors in section 2.1 works in general only up to dimension 3. Regarding magnitudes (section 2.3), we must point out, that non-Euclidean vector spaces may have (isotropic) null-vectors, which square to zero. Regarding angles and exponential functions (section 2.5) the corresponding section of [Hestenes, D. (1986), New Foundations for Classical Mechanics, Reidel, Boston] should be read alongside for clarity. Chapter 3 carefully relates a *plane of vectors* with a 1:1 corresponding *spinor (complex) plane* or rotation-dilations in the geometric algebra of the Euclidean plane.

The *dual* of a multivector is defined in section 4.1 as the product with the unit pseudoscalar, Hestenes uses instead the inverse of the unit pseudoscalar. Regarding section 4.2 we point out, that in the geometric algebra of $\mathbb{R}^3$ the dual of *any* unit vector squares to -1. Regarding chapter 5, it may be good for clarity and deeper understanding to study the corresponding sections of [Doran, C., Lasenby, A. (2003), Geom. Alg. f. Phys., CUP] alongside chapter 5. The *rotor* of a Lorentz rotation can only be factorized (equation 5.54) under special conditions (like the boost and spatial rotation factorization of section 9.1.5). Regarding the relationship of vectors and spinors (quaternions, rotors) expounded in chapter 6, we emphasize that the underlying vector space is a linear subspace of a geometric algebra, but not a subalgebra.

Part II may be read independent of part I, because chapters often begin with a summary of related material from part I. Chapter 7 reviews the *Maxwell equations*, including their coupling to *gravitational fields* in Riemann space-time and in Einstein-Cartan space-time. The latter includes torsion. Then the complete unification of the Maxwell equations in Space-Time Algebra (STA) [Hestenes, D. (1966), Space-Time Algebra, Gordon and Breach, New York] is explained.

Using this setting, chapter 8 discusses (polarized) electromagnetic waves, and Maxwell equations in spinor (quaternion) form, the latter being originally considered by Maxwell himself. Care must be taken of the signs on the right hand sides of equations 8.42 to 8.45. Section 8.5 compares Majorana and Weyl's form of the Dirac equation with the spinor form of the Maxwell equations. Chapter 9 shows how to express the *Dirac equation* in STA [Hestenes, D. (1966), Space-Time Algebra, Gordon and Breach, New York]. The wave function spinors turn out to be nothing but proper Lorentz transformations (STA rotors V) with scalar-pseudoscalar amplitudes. Their magnitude is the probability density. This induces a new geometric interpretation of spin based on a (local) spin bivector $\sim V\gamma_1\gamma_2\tilde{V}$. Chapter 9 ends with a detailed discussion of *neutron interferometry* in the framework of STA, of the coupling of *spin density* to *contorsion*, and charge conjugation. STA completely *unifies* the concepts of internal and external spaces.

Chapter 10 opens with a discussion of *quantum gravity* (Einstein-Cartan) and geometric algebra. Torsion Q and curvature R are regarded as independent conjugate geometric variables, subject to the uncertainty relation $[Q, R] = \gamma_1\gamma_2 L_{Pl}^{-3}$, where L_{Pl} is the Planck length. For this form of commutation relation six bivector gauge conditions must be fulfilled. Next a gauge invariant Lagrangian density is constructed. (A gauge field approach was also taken by Lasenby, Doran and Hestenes, e.g. in [Hestenes, D. (2005), Gauge theory gravity with geometric calculus, Found. Phys. 35(6)].) The gauge potential connection bivectors include Ricci connections and contorsion Q. The exterior covariant derivative of Q yields the curvature trivector (Bianchi identity). The Lagrangian includes terms quadratic in R and Q, and an Einstein-Hilbert term. Classical gravity results for week fields and canonical quantum gravity for strong fields. The work concludes with a discussion of *spin fluctuations* at the Planck scale and their implications for the quantization of space-time, zitterbewegung, nucleon mass, and Compton wavelength.

Comments to the MR Editors: It was not easy to read this book, because it contains a number of technical errors. In addition, many symbols often change their fonts. The English language can be understood, but could be easily improved. The overall quality of the figures is also lacking. I tried to not be negative about the book, but there were a number of irritating mistakes, which I just had to point out. I hope that these hints, together with the background references, will prevent readers from misjudging the book purely on technical grounds. Because the physical concepts introduced are indeed very interesting and derserve attention.