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Review text:

This paper addresses students, researchers and engineers in robotics and computer vision. It reviews some basic theory and gives several examples of applications.

First the basic notions of Clifford geometric algebra are reviewed. The vector space basis of section 2.1 needs to be *orthonormal*. We remark that the authors work with Hestenes and Sobczyk's inner product [Hestenes D., Sobczyk, G. (1984), Clifford Algebra to Geometric Calculus, Reidel, Boston], which causes exceptions for scalars. Contraction may be preferable; for a discussion see appendix B of [Dorst L., Fontijne D., Mann S. (2007), Geometric Algebra for Computer Science, The Morgan Kaufmann Series in Computer Graphics, Elsevier, Amsterdam.] The scalar product (p. 986) is the scalar part of the geometric product. The *signature* of an r-vector (p. 986) is related to, but not identical with the quadratic form specification of equ. (2). A geometric algebra is *generated* (and not *spanned*) by the geometric product of vectors of a vector space (p. 986). *The even subalgebra* is the set of *all* even grade elements of a geometric algebra, not only *a subalgebra* generated by multivectors of even grade.

Care must be taken with the definition of *dual* multivectors in degenerate geometric algebras. In section 2.2 the authors seem to use the vectors σ_k and $e_k, k \in \{1, 2, 3\}$ interchangeably. There seems to be made no distinction between *conjugate* and *reverse* of a multivector. The intersection (meet) of two planes uses the set theory intersection symbol on page 1000, but not in the definition of equ. (26). Care needs to be taken with this definition, because the dual is only to be taken with respect to the join (union) of the two factors [Dorst L., Fontijne D., Mann S. (2007), Geometric Algebra for Computer Science, The Morgan Kaufmann Series in Computer Graphics, Elsevier, Amsterdam.]

In section 3 a brief introduction is given to so-called conformal geometric algebra. For more details [Dorst L., Fontijne D., Mann S. (2007), Geometric Algebra for Computer Science, The Morgan Kaufmann Series in Computer Graphics, Elsevier, Amsterdam.] is a good reference. In section 3.1 only projective geometric algebra is treated. Here the vectors e and e_0 for the origin are obviously used interchangeably. The blade of equ. (36) can be computed from $(a_1 + e_0) \wedge (a_2 + e_0) \wedge (a_3 + e_0)$. In equ. (38) the second factor $\mathbf{A}^k$ needs to be replaced by the moment M^k only, otherwise a term linear in e_0 arises. Equation (39) is not explicit enough to calculate actual distances. For that the reader may e.g. refer to [Hitzer, E. (2005), Conic Sections and Meet Intersections in Geom. Alg. In H. Li, P. Olver, G. Sommer (eds.), Comp. Alg. and Geom. Alg. with Appl., Springer, LNCS 3519, pp. 350-362.]

Section 4 shows how inversions at a sphere, reflections at planes, general rotations and translations can be implemented in conformal geometric algebra as Clifford (Lipschitz) group versor transformations. In the simplest cases of inversions at a sphere (and reflections at a plane), the versor is simply the 5D vector in $\mathbb{R}^{3+1,1}$, that specifies the sphere (or plane). For clarity we recommend to read chapter 13 of [Dorst L., Fontijne D., Mann S. (2007), Geometric Algebra for Computer Science, The Morgan Kaufmann Series in Computer Graphics, Elsevier, Amsterdam.] alongside sections 4.2 and 4.3.

Section 5 briefly reviews conics (curves and surfaces), ruled surfaces, cycloidal curves and helicoids (screw transformation of straight line) and the Plücker conoid in conformal geometric algebra. For a good overview see Table 2.1 of [Rosenhahn, B., Perwass, C., Sommer, G., (2002), Pose estimation of 3D free-form contours. Tech. Rep. 0207, Univ. of Kiel.] Section 5.2 implements conics as intersections with parametric ruled surfaces. For linear object oriented implementations see [Perwass, C., Förstner, W. (2006), Uncertain geometry with circles, spheres and conics. In Geom. Prop. from Incomplete Data, R. Klette et al (eds.), Comp. Imaging and Vision, Vol. 31, pp. 23-41, Springer, Heidelberg; Hitzer, E. (2005), Conic Sections and Meet Intersections in Geom. Alg. In H. Li, P. Olver, G. Sommer (eds.), Comp. Alg. and Geom. Alg. with Appl., Springer, LNCS 3519, pp. 350-362.]

Section 6 on applications shows how based on the approach developed in the previous sections a robot equipped with stereo cameras can follow a welding line or a ruled surface 3D curve, and a robot arm grasping algorithm.

Comments to the MR Editors: The theoretical part contains a few errors and imprecise statements. This may be quite confusing for the reader. In some cases I have corrected that in the review in others I simply recommend other references for "clarity".