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Short title: On unique parametrization of the linear group GL(4, **C**) and its subgroups by using the Dirac algebra basis.

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Review text:

This well written, easy-to-read and concise review of ways to parametrize the linear group $GL(4, \mathbb{C})$ and its subgroups is very suitable for students and researchers alike in both mathematics and theoretical physics. The paper includes a very extensive historical review (271 references) of a century of research on spinors from G. Darboux's work (1905) to A. Gsponer and J.P. Hurni's bibliographies on quaternions in mathematical physics (2006). The authors are from the Feodorov school.

First the $GL(4, \mathbb{C})$ multiplication is parametrized in terms of Dirac matrices (scalars = grade 0, . . . , pseudoscalars = grade 4). Then odd grades (vectors = grade 1 and pseudovectors = grade 3) are excluded arriving at a definite way to parametrize the spinor covering $SO(4, \mathbb{C})$ of the complex Lorentz group. In this context Feodorov's parametrization of complex orthogonal Lorentz transformations is put in context and compared with parametrizations by complex tensors, complex bivectors, two complex 4-vectors, two complex 3-vectors (two variants), complex Euler angles, and complex curvilinear coordinates. Restrictions (transitions) to the real Lorentz group $O(3, 1; \mathbb{R})$, the real orthogonal group $O(4; \mathbb{R})$, and to the real orthogonal group $O(2, 2; \mathbb{R})$ are explained. Euler's complex angles lead to a nice factorization of the 2×2 blocks of the quasidiagonal complex Lorentz group spinors covering matrices. This further leads to the elegant factorized structure of covering 4-spinor transformations and 4-vector transformations due to Einstein and Meyer's (1932) theory of semi-vectors.

The authors explain with the help of quaternions an algorithm for explicitly determining the 8 parameters of a complex Lorentz group transformation matrix. With a similarity transformation to an isotropic basis (compare Newman-Penrose), the Lorentz matrix consists of 16 single binomial entries.

Next the authors show that there exist only two different types of pure spinor representations of the covering group of the complex Lorentz group obtained by adding parity transformation P and time reversal T. Within the limits of the partly extended spinor groups (improper orthochronous and proper non-orthochronous) representations are related by a similarity transformation, leaving only one representation. These results are obtained based on the representation theory for the *spinor covering* of the orthogonal group in view of the historical problem of intrinsic parities of fermions. Finally an explicit general 17 parameter form of Majorana spinor bases in relation to some fixed Weyl spinor basis is established.