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Review text:

This paper is suitable for graduate students and researchers with basic knowledge of gauge field theory, special relativity, matrix algebra and quantum mechanics.

After a general introduction (Section 1), Section 2 introduces the (Clifford) geometric algebra $G_{4,1}$ ($Cl_{4,1}$) of the Minkowski space $\mathbb{R}^{4,1}$, where the three physical space vectors and an extra dimension for proper time τ square to $+1$, while the time vector squares to -1. The isomorphism to the complex 4×4 matrix algebra $M(4, \mathbb{C})$ is employed for representing the five basis vectors of $\mathbb{R}^{4,1}$. Exponentials of bivectors (rotors) are explained.

A 5D reciprocal frame (basis) is introduced in order to define a vector derivative and a Laplacian, both of which are split into $4 + 1$ or $1 + 3 + 1$ parts.

In Section 3 it is shown how from the $2^5 = 32$D basis of $G_{4,1}$ a set of 4 mutually annihilating idempotents can be defined, which serve in turn to construct 8 $SU(3)$ generators, whose 4×4 matrix representation is given explicitly, embedding 3×3 Gell-Mann matrices. Along the same lines 7 additional $SU(4)$ generators are developed, and a second set of generators for a second $SU(3)$ group, independent from the first $SU(3)$ group, is introduced.

Section 4 introduces plane wave solutions to the $(4+1)$D monogenic condition (zero vector derivative). This becomes a Dirac equation after the angular frequency is interpreted as energy, and the fourth component of the wave vector as mass, and with a certain choice of Dirac subalgebra in $G_{4,1}$. By enhancing the vector derivative to a gauge derivative, electromagnetism is incorporated. Then

the possible algebraic set of 16 ”imaginary units” (geometric algebra square roots of -1) in the plane wave solution is discussed, determined by quantum number coefficients $a_\mu, \mu = 1, 2, 3, 4$. The a_μ can be converted to $SU(4)$ generator coefficients. Based on the a_μ a tentative association (for 12 of 16) with the quantum numbers of electric charge and isospin follows for two families of elementary particles.