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Review text:

This short well-written paper is suitable for students and researchers with basic knowledge of special relativity, hypercomplex algebra and differential geometry.

First biquaternions ($\mathbb{H}\otimes\mathbb{C}$) represented in $M(2,\mathbb{C})$ are embedded in $\mathbb{C}^4$ with complex coordinates $z_\mu \in \mathbb{C}, 0 \leq \mu \leq 3$. The internal 3 complex parameter transformations $SO(3,\mathbb{C})$ preserve biquaternion multiplication. Trace z_0 remains invariant, z_1, z_2, z_3 undergo complex rotations, and $z_0^2 - \sigma$, $\sigma = z_1^2 + z_2^2 + z_3^2$ are 2 complex invariants. The real (metric Minkowski space) interval $S^2 = \sigma\sigma^* = T^2 - |X|^2 \geq 0$ is reparametrized by two pure quaternions $\mathbf{p}, \mathbf{q}$ (real and imaginary parts of z_1, z_2, z_3) to (positive definite) physical time $T = |\mathbf{p}|^2 + |\mathbf{q}|^2$, and physical Euclidean radius vector $X = 2\mathbf{p} \times \mathbf{q}$. $SO(3,\mathbb{C})$ transformations describe now proper Lorentz transformations.

Next increments δX and δT are discussed in terms of increments $\delta\mathbf{p}, \delta\mathbf{q}$. δT appears irreversible, any closed point particle $\mathbf{p}(\lambda), \mathbf{q}(\lambda), \lambda \in \mathbb{C}$ path has nonzero physical time T, perhaps related to time delay. Then the restoration of the "hidden" complex $\delta\mathbf{p}, \delta\mathbf{q}$ structure from Minkowski space-time $\delta X, \delta T$ is discussed, mapping the complex null cone $\delta\sigma = 0$ to real light cone $\delta S^2 = 0$. Only a common 1 parameter rotation with axis δX remains. In general there is a "hidden" geometric phase $\alpha(\delta\mathbf{p}, \delta\mathbf{q})$ invariant under Lorentz transformations, α possibly related to quantum interference. If α is fixed the angle θ between $\delta\mathbf{p}$ and $\delta\mathbf{q}$ can be computed depending on velocity v and on α with extremes $\theta = 0$ and $\theta = \pi$, which might be related to quantum spin.

Finally "hidden" algebraic dynamics in complex-quaternionic space and its

image in Minkowski space-time is discussed using techniques of differential geometry. It is found that the real Minkowski interval is the modulus of the complex proper time of an elementary observer. For restoring time order the introduction of an evolution curve is suggested. Variations of the α phase could be related to quantum uncertainty and interference.