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Short title: Unitary congruence and unitary similarity for quaternion matrix.

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Review text:

This short paper is suitable for students and researchers with basic knowledge of complex and quaternion algebra and matrix algebra. It deals with finite $n \times n$ matrices A with complex and quaternion entries.

In the introduction the notions of transpose, conjugate, transpose conjugate ($A \to A^*$), and unitary matrices ($U^*U = UU^* = I$) are defined. This is followed by definitions of (unitary) similarity. If i, j, k are the quaternion units, then a quaternion matrix is represented complex as $A = A_1 + A_2 i + (A_3 + A_4 i)j$, where $A_l \in \mathbb{R}^{n\times n}, l = 1, 2, 3, 4$. Special notions are the j-transpose $A^J = -jA^*j$, and the j-conjugation $\widetilde{A} = -jAj$. The j-transpose serves to define unitary congruence: $A, B, U, P \in \mathbb{H}^{n\times n} : B = U^J AU$, U unitary. Consimilarity is defined by $B = \widetilde{P}^{-1}AP$. Every Hermitian quaternion matrix $A \in \mathbb{H}^{n\times n}$ can be diagonalized (with a unitary matrix $U \in \mathbb{H}^{n\times n}$) with real eigenvalues.

The first section treats the unitary congruence of quaternion matrices including finite sets of simultaneously unitarily congruent and consimilar matrices. It is found that every matrix $A \in \mathbb{H}^{n\times n}$ is unitarily congruent to an upper triangular matrix. The relationship of complex unitary congruence and unitary congruence (as quaternion matrices) is explored.

The second section deals with the unitary similarity of quaternion matrices $A, B, U \in \mathbb{H}^{n\times n} : B = U^*AU$, where U is unitary.

The third section deals with the unitary congruence for special quaternion matrices: J-symmetric matrices $A^J = A$, which are unitarily congruent to diagonal matrices with real positive (or zero) eigenvalues. A corresponding property is established for J-skew-symmetric matrices $A = -A^J$, which are unitarily congruent to diagonal matrices with imaginary eigenvalues (real multiples of j). Symmetric $A = A^T$ and skew-symmetric $A = -A^T$ quaternion matrices are also treated. Finally it is shown that conjugate normal quaternion matrices A (with $A^*A = \widetilde{AA^*}$) can be diagonalized with complex eigenvalues $a_l + b_l j$, with real $a_l \geq 0$, and real b_l, $l = 1, \dots, n$.

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