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Review text:

This paper is suitable for students and researchers with an interest in cumputer algebra computations in Clifford and Grassmann algebras.

It describes the basic definitions and procedures of the CLIFFORD package for MAPLE (http://math.tntech.edu/rafal/) for defining a Clifford algebra dependent on an arbitrary bilinear form. Clifford algebras $Cl_{p,q}, p + q = n \leq 9$ have been precomputed and the clidata procedure returns the following information list: (1) isomorphism to a real $\mathbb{K} = \mathbb{R}$, complex $\mathbb{K} = \mathbb{C}$, or quaternionic $\mathbb{K} = \mathbb{H}$, matrix algebra. (2) Dimension of the isomorphic matrix algebra. (3) Information whether the algebra is simple or semi-simple. (4) Precomputed idempotent f. (5) Basis in spinor ideal $S = Cl_{p,q}f$. (6) Subalgebra of $Cl_{p,q}$ isomorphic to $\mathbb{K}$. (7) List of generators of S, viewed as right module over $\mathbb{K}$.

Section 3 introduces the definition of a Clifford algebra over an arbitrary bilinear form via Chevalley deformation and Clifford map, and the relevant procedures in CLIFFORD.

Section 4 shows how to work with Lounesto's [P. Lounesto, *Clifford Algebras and Spinors*. 2nd ed. (Cambridge University Press, Cambridge, 2001).] dotted and undotted Grassmann bases in CLIFFORD, which is of interest in quantum field theory and representation theory. In the dotted wedge product definition the antisymmetric part F of the bilinear form $B = g + F$ is split off and contraction terms dependent on F are added to the undotted wedge product.

Sections 5 and 6 give examples computed with CLIFFORD for Clifford product operations showing the associativity of the dotted wedge product, and the reversion anti-automorphism for both the dotted and undotted wedge product, respectively.

Section 7 shows how to compute with Clifford algebra spinor representations, giving several concrete examples. Section 8 shows how to compute with two different scalar products in spinor ideals, one involving reversion only, the other involving conjugation (a composition of grade involution and reversion).

Section 9 shows with concrete examples how to compute continuous families of Clifford algebra idempotents with CLIFFORD.

Section 10 demonstrates how to compute in CLIFFORD with 2×2 Vahlen matrices, containing Clifford algebra entries that meet certain special conditions. In higher dimensions sense preserving conformal mappings are restrictions of Möbius transformations and may be represented by Vahlen matrices (including transversions or special conformal transformations).

Section 11 gives a thorough introduction (with an explicit example) on how to embed any matrix A into a matrix algebra isomorphic to a Clifford algebra, and subsequently how to compute a singular value decomposition fully in the language of Clifford algebra. A suggested anti-automorphism in Clifford algebra, which gives the transposition of the corresponding spinor representation has also proved useful in the context of Clifford algebra neural networks [S. Buchholz, E. Hitzer, K. Tachibana. *Coordinate independent update formulas for versor Clifford neurons.* Proc. of Joint 4th Int. Conf. on Soft Comp. and Intel. Sys., and 9th Int. Symp. on Adv. Intel. Sys., Sep. 2008, Nagoya, Japan, pp. 814 - 819 (2008)].

The appendix provides the pseudocode of several important CLIFFORD products and procedures. It is to be hoped for that in the future the author may extend his excellent treatment to a full CLIFFORD package based textbook for Clifford algebra, amply illustrated by concrete CLIFFORD examples.