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Review text:

This paper is suitable for graduate students and researchers with an interest in Grassmann algebra and Clifford's geometric algebra applied to modern cosmological problems.

It begins by giving a brief introduction to Clifford's (associative) geometric algebra, which is based on Grassmann's exterior algebra and Hamilton's quaternions. Importantly, in geometric algebra all multivector entities are reflected at hyperplanes with the same (covariant up to sign) formula. This leads to a general covariant rotor description of rotations of all multivectors in all dimensions and signatures. It allows to describe electromagnetism with a single unified multivector Maxwell equation, of the simplest possible form - viewed in geometric algebra. This extends to a multivector form of the Dirac equation, with deep geometric insights into quantum mechanics. A right side rotor gauge freedom incorporates electromagnetism and electroweak transformations. A left side rotor gauge leads to a gauge theory of gravity, locally reproducing general relativity, facilitating computations (e.g. fermion energy levels around a black hole).

Section 3 introduces conformal geometric algebra (CGA) with an additional 2d Minkowski type plane representing the origin and infinity. Basic geometric entities become simple multivectors in CGA. All covariant combinations have a clear geometric meaning. This extends to higher dimensional models of Euclidean, hyperbolic and spherical spaces. It leads to a novel representation of de Sitter space and its boundary structure, with interesting physical consequences when embeding a big bang structure.

Section 4 models a slightly closed universe within this new approach. A condition for the total elapse time (big bang to infinite future) is found. A predicted critical path diagram is discussed in relation to cosmic microwave background (CMB) data. For a consistent inflationary model, a simple scalar field is introduced. The resulting field equations are discussed with a power series expansion. One parameter relates inflation with the cosmological constant, the other governs the curvature, with interesting theoretical CMB predictions.

Section 5 comments on the possiblities of new CMB structure data (e.g. by the Planck Satellite) to give information about string theory at energy scales 10^{12} higher than achievable at the Large Hadron Collider (LHC).

Section 6 elegantly discusses Bianchi models in CGA, using only a 1d-up approach (instead of 2d-up of Section 3). Examples for Biachi IX (very natural coordinatisation) and VII_h are given. For Biachi IX the Killing vectors found geometrically describe screw motions, independently found group theoretically by A. Pontzen, Phys. Rev. D **79**(10), 103518ff. (2009). Exact Bianchy IX computations (in CGA) can (in principal) fit CMB power spectra and large scale structure data. Of further current interest is a biaxially symmetric Bianchi IX model with scalar field, which gets smoothly through the big bang, with no singularities.