This is a review text file submitted electronically to MR.

Reviewer: Hitzer, Eckhard M. S.

Reviewer number: 36883

Address:

Department of Applied Physics
Fukui University
3-9-1 Bunkyo
Fukui, Fukui 910-8507
JAPAN
hitzer@mech.u-fukui.ac.jp

Author: Baylis, W. E.; Cabrera, R.; Keselica, J. D.

Short title: Quantum/classical interface: classical geometric origin of fermion spin.

MR Number: 2737603

Primary classification: 81R25

Secondary classification(s): 15A66, 81P40, 83A05, 81P15

Review text:

This paper is suitable for students and researchers with basic knowledge of special relativity and quantum mechanics.

Section 2 introduces the algebra of physical space (APS) as the Pauli algebra, isomorphic to Clifford's geometric algebra $Cl(3,0)$ of three-dimensional Euclidean space with orthonormal vector basis $\{e_1, e_2, e_3\}$. Rotors (exponentials of bivectors) R give a spinor representation of rotations. Rotating a rotor by 2π leads to a factor of -1. Combining rotors and projectors, e.g. RP_3, $P_3 = (1 + e_3)/2$, leads to irreducible spinors. The APS can also be generated from a pair of annihilation $a = e_1P_3$, and creation $a^\dagger = P_3e_1$ operators.

In Section 3 paravectors, sums of scalars and vectors, are introduced. With the help of the Clifford conjugate (symbol overbar) paravectors get a quadratic form equivalent to the metric of Minkowski spacetime. Exponentials of biparavectors yield Lorentz rotors. Multiparavectors mix neighboring vector graces (of $Cl(3,0)$) and lead to a 16-dimensional algebra. In this framework classical eigenspinors Λ (Lorentz rotors relating rest and lab frames) for relativistic dynamics, equations of motion for these eigenspinors, the electromagnetic field as spacetime rotation rate leading to: a spinor form of the Lorentz force, Maxwell equations and a local gauge transformation representing spin as a rest-frame rotation.

Section 4 dicusses the Dirac equation in APS. Let ρ be the particle density in the rest frame. The eigenspinor of Section 3 leads to $\psi = \rho^{1/2}\Lambda$, the Dirac spinor in the Weyl representation becomes $\frac{1}{\sqrt{2}}(\psi P_3, \bar{\psi}^\dagger P_3)$. The minimal left ideals $(APS)P_3$, $(APS)\bar{P}_3$ give left and right chiral ideals of APS. This is followed by a discussion of De Broglie waves (after a boost from the rest frame) and spin interaction, magnetic moment ($g = 2$), spin as intrinsic physical rotation, and mass originating from a magnetic moment of spin accelerated in a static inhomogeneous magnetic field. Large and small components of ψ are discussed yielding the standard Dirac-Pauli representation. Formulating differential operators and commutation relations in APS clarifies the relation of imaginary unit $i = e_1e_2e_3$ and spin as biparavector $S = \Lambda e_1\bar{e}_2\bar{\Lambda}$: $i\psi e_3 = -S\psi$. In the proposed model Pauli's objection to considering time as an operator in $[H, t] = i\hbar$ is resolved by regarding phase factors $e^{i\varepsilon t}$ only as projections of the actual modification of a particle rotation by fields (e.g. a magnetic field). The APS thus reveals the geometric significance of the Dirac equation.

Section 5 studies spin distributions, spin densities, state filters, pure and orthogonal states, and state expansions. Next it is shown how in the APS model of fermion spin the quantum result of the Stern-Gerlach experiment is obtained, as well as uncertainty of spin measurement, violation of the Bell inequalities, and entangled systems.

The conclusions tries to summarize as to what is described in the classical *elmentary* APS systems, versus what could still be regarded an essential nonclassical core of quantum behavior, like the very existence of quanta, Born's probability current interpretation, the measurement problem and the wave-function collapse.