

This is a review text file submitted electronically to MR.

**Reviewer:** Hitzer, Eckhard M. S.

**Reviewer number:** 36883

**Address:**

Department of Applied Physics  
Fukui University  
3-9-1 Bunkyo  
Fukui, Fukui 910-8507  
JAPAN  
hitzer@mech.u-fukui.ac.jp

**Author:** Dereli, Tekin; Kocak, Sahin; Limoncu, Murat

**Short title:** Degenerate spin groups as semi-direct products.

**MR Number:** 2737605

**Primary classification:** 15A66

**Secondary classification(s):** 11E88

**Review text:**

This paper is suitable for students and researchers with basic knowledge of algebra and group theory. It deals with *degenerate Clifford algebras*  $Cl_{p,q,r}$  over  $n$ -dimensional vector spaces with symmetric bilinear forms,  $n = p + q + r$ ,  $e_k^2 = +1, 1 \leq k \leq p$ ,  $e_k^2 = -1, p + 1 \leq k \leq p + q$ ,  $e_k^2 = 0, p + q + 1 \leq k \leq n$ . Proposition 2.1 states that the following subset of  $Cl_{p,q,r}$  is a group under Clifford multiplication

$$S_{p,q,r} = \{s\gamma_1 \dots \gamma_{p+q}(1 + G) \mid s \in Spin(p, q), \gamma_i = 1 + e_i \sum_{l=1}^r c_{il} f_l, G \in \Lambda(f)\},$$

with  $1 \leq i \leq p + q, c_{il} \in \mathbb{R}$ ,  $\Lambda(f)$  the span of all blades formed only with the degenerate vectors  $\{f_1 = e_{p+q+1}, \dots, f_r = e_n\}$ . The *degenerate spin groups* are defined as the quotient groups

$$Spin(p, q, r) = S_{p,q,r} / \Delta, \quad \Delta = \{1 + G \mid G \in \Lambda(f)\}.$$

The main theorem states that  $Spin(p, q, r) \simeq Spin(p, q) \rtimes_{\tilde{\rho}} \mathbb{M}(p + q, r)$ . The isomorphism maps  $[s\gamma_1 \dots \gamma_{p+q}(1 + G)] \mapsto (s, (c_{il}))$ ,  $c_{il} \in \mathbb{M}(p + q, r)$ ,  $\mathbb{M}(p + q, r)$  the additive group of  $(p + q) \times r$  matrices,  $\tilde{\rho} : Spin(p, q) \rightarrow Aut(\mathbb{M}(p + q, r))$ ,  $s \mapsto \tilde{\rho}(s)(A) = \rho(s)A$ ,  $\rho(s)$  the  $SO(p, q)$  rotation matrix associated with  $s \in Spin(p, q)$ ,  $A \in \mathbb{M}(p + q, r)$ . All proofs are done in an elementary step by step manner.